Technical Notes on Sage's 23-tone Test

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1 Introduction

23-tone is an IEEE-743 recommended test for rapid characterization of a communication channel in terms of its linear transmission response (attenuation and envelop-delay distortions), nonlinear distortions (second and third order intermodulation distortions), signal-to-total-distortion and signal-to-noise ratios. Recently, Sage also added the channel capacity measurement.

Detailed 'specifications' of 23-tone test are covered in section 8.6 of IEEE 743. This paper covers the actual implementation in Sage's platforms, the commonly seen numbers across various test lines, some peculiarities of the 23-tone test and the interpretation of each measurement. This paper intends to help our test engineers, SW developers and marketing fellows gain more quantitative understanding of this important test. The last contains detailed DSP algorithms, which are useful for future software developers to maintain and port the code.

2 Definitions of the Measurements

2.1 Physical definition of each measurement

23-tone test, as implemented in Sage's platforms, measures the following parameters:

1. Composite power: The total averaged power of the 23-tone signal measured at the receiving end. This definition implies that only flat-gain change and band-limiting filtering can affect the composite power. Pure additive noise should not affect the composite power measurement, neither should IMDs, unless there is significant amount of third order IMD.

2. Attenuation distortion: The measured attenuations (losses) reflect the under-test-channel's amplitude response characteristics (power spectrum) at 23 equally spaced frequency points. These frequencies are $F_n = (10n+13)125/8, n = 0, 1, 2, \ldots, 22$. Uneven attenuation on different frequency components causes dispersion on the transmitted signal. For voice, this means muffling effect (or loss of clarity). For modem signal, this means inter-symbol-interference. For example, if one transmits a symmetric pulse (half-cosine or Gaussian pulse) down the line with uneven attenuation distortion, the pulse shape will become asymmetric at the receiving side.

3. EDD: EDD (Envelop-delay-distortion) measures the phase response characteristics of a channel under test. The EDDs are calculated as group delays (negative slope of phase response curve, i.e., negative phase difference over angular frequency difference) at 22 equally spaced frequencies. These 22 frequencies are the middle points of the 23 frequencies where attenuations are measured. If a channel has linear phase response (therefore, a constant slope), the

EDDs will be constant (and normally reduced to zeros), meaning all frequency components are experiencing the same amount of delay, and therefore, no signal dispersion occurs. But a real analog system (circuitry or transmission line) will always have some amount of nonlinear phase response, causing the EDDs to be non-constant, meaning different frequency components are experiencing different amount of delays. In time-domain, the effect of EDD is to make a symmetric Gaussian-type pulse asymmetric at the receiving end.

4. IMD_2: The ratio of the 23-tone signal power to the power of its second order harmonics within a selected band. Both IMD2 and IMD3 (defined below) measure the non-linear distortions of the channel. Counter-intuitively, the higher the IMD numbers, the less the non-linear distortion.

5. IMD_3: The ratio of the 23-tone signal power to the power of its third order harmonics within a selected band.

6. SNR: Signal-to-noise ratio. The noise term includes only the additive non-correlated circuit noise (quantization noise and ambient electronic noise etc). It does not include the IMD terms, which can be considered as correlated interference. As usual, the noise power is measured through a filter (D-filter, in this case).

7. STD: Signal-to-total-distortion. The total distortion includes both the additive non-correlated circuit noise and the IMD interference. That is to say, total distortion includes all 'garbage' signals except the original 23-tone components. As in SNR, the total distortion is measured through a D-filter.

8. Channel capacity: Maximal data rate that can be supported by a channel under test. When transmitting above this limit, data errors will occur. When transmitting below this limit, theoretically (through ingenious modulation and error encoding schemes), the data error probability can be made infinitely small. Notice that the measured channel capacity only applies to voice band carrier modulated modem (such as V.34). It does not apply to the special PCM V.90 modem, because V.90 modem does not see the quantization noise as seen by the the 23-tone test. Neither can it measure the intrinsic data rate capacity that a local loop can carry. The upper limit of the measured capacity is set to 64 kbps, the Nyquist rate (Maximum symbol rate (8KHz) times bits per symbol (8b)). The intrinsic data capacity of a twisted pair is well above 1 Mbps, as demonstrated by DSL modem and 10base-T LAN network.

2.2 Mathematical definitions

From system point of view, a communication channel introduces three types of distortions to a test signal, linear distortion, nonlinear distortion, and additive noise. Assume the test signal is $x(t)$, which is a 23-tone signal in this case. After going through a channel under test, the received signal becomes [1]:

$$\begin{aligned} s(t) &= s_{lin}(t) + [s_{IMD2}(t) + s_{IMD3}(t)] + s_{noise}(t) \\ &= x(t) \otimes h_1(t) + [k_2 x^2(t) + k_3 x^3(t)] \otimes h_2(t) + w(t) \otimes h_3(t) \end{aligned}$$

[1] We use continuous signal model for the ease of discussions here. In real implementation as shown in the last section, the algorithm works on discrete samples.

where the first term in rightmost side of the equation represents linear distortion, the second term represents second and third order intermodulation distortion and the third term is additive non-correlated noise. $\otimes$ stands for convolution operation.

As the equation implies, $h_1(t)$ is the impulse response of the channel that the original 23-tone signal has gone through. $h_2(t)$ is the impulse response of the channel that the IMD components have gone through. $h_3(t)$ is the impulse response of the channel that the white noise has gone through. All these three impulse responses are not necessarily the same. If the IMDs and white noise are both introduced at the transmitting side, then they are all equal. But if the IMDs are introduced in the middle of the system, and the noise is introduced at the receiving side, then they will all be different.

$h_1(t)$ carries information about the attenuation and envelop delay distortions of the channel under test. $s_{IMD}(t)$ carries information on IMDs, and $s_{noise}(t)$ carries information on $\frac{S}{N}$ and $\frac{S}{TD}$. The goal of the 23-tone measurement algorithm is to extract each component from the received signal $s(t)$, and separate each component from each other. The algorithm achieves this goal through Fourier transform and time-domain and power-spectral-domain averaging. Time-domain averaging removes (or reduces) the non-correlated noise term $s_{noise}(t)$. The original 23-tone components are separated from the IMD components in frequency domain as they occupy different frequency bins. Detailed algorithms are shown in the last section.

To mathematically define each measurement, we assume the frequency response of $h_1(t)$ is $A(f)e^{j\phi(f)}$, and define the average power of a signal component as

$$Power(g(t)) = \frac{1}{T}\int_0^T |g(t)|^2 dt$$

, and $f_n = (10n + 13)125/8, n = 0, 1, \ldots, 22$. Then the definition of each measurement becomes:

1. Composite power:

$$CP_{dBm} = Power(s_{lin}(t)) = XmtPower_{dBm} + 10\log[\frac{1}{23}\sum_{n=0}^{22}|A(f_n)|^2]$$

2. Attenuations (losses):

$$Loss_{dB}(n) = -10\log(|A(f_n)|^2), n = 0, 1, \ldots, 22$$

3. EDDs:

$$dt(n) = -\frac{1}{2\pi}\frac{\phi(f_{n+1}) - \phi(f_n)}{f_{n+1} - f_n}, n = 0, 1, \ldots, 22$$

4. IMD2:

$$IMD2_{dB} = 10\log(\frac{Power(s_{lin}(t))}{Power(s_{IMD2}(t)})$$

5. IMD3:

$$IMD3_{dB} = 10log(\frac{Power(s_{lin}(t))}{Power(s_{IMD3}(t)})$$

6. STD:

$$\frac{S}{TD} = 10\log(\frac{Power(s_{lin}(t))}{Power(D(t)\otimes[s_{IMD2}(t) + s_{IMD3}(t) + s_{noise}(t))})$$

   where $D(t)$ is the D-filter specified in IEEE 743. The equation means that the total distortion is measured through a D-filter.

7. SNR:
$$\frac{S}{N} = 10\log(\frac{Power(s_{lin}(t))}{Power(D(t) \otimes s_{noise}(t)})$$
This means, the noise is measured through a D-filter.

8. Channel Capacity:
$$C_{bps} = \int_0^{4000} \log_2(1 + \frac{S}{TD}(f))df$$

3 Commonly Seen Numbers

3.1 Clean analog circuits:

When testing through a 'clean' dry circuit or through 'analog' TAS (such as two 930s or 923 to 356E+), the following numbers are commonly seen:

1. *Attenuations:* The attenuation curve should indicate a normal band-passing nature. That is, it should be relatively flat near 1KHz and roll off below 300 Hz and above 3300 Hz.

2. *EDDs:* Roughly speaking, the EDD curve should look like an upside down image of the attenuation curve [2]. That is, in the middle band near 1KHz, the EDDs should be relatively flat and close to zeros. It increases at the low and high frequency edges.

3. *IMD2:* $IMD_2 > 45$ dB.

4. *IMD3:* $IMD_3 > 45$ dB.

5. *STD:* $\frac{S}{TD} > 45$ dB.

6. *SNR:* $\frac{S}{N} > 45$ dB.

7. *Channel Capacity:* $C > 34$ Kbps.

3.2 Clean PCM channel:

When testing through a PCM channel and both ends are digitally connected (such as the case with two 930s or 950 to 950), then the measurements will be dominated by the PCM quantization noise. The following numbers are commonly seen.

1. *Attenuations:* Attenuation curve should be flat across the whole voice band (0 ± 0.1dB).

2. *EDDs:* EDD curve should be flat across whole band ($0.01 \pm 0.01 ms$).

3. *IMDs:* $IMD_2 > 45$ dB and $IMD_3 > 45$ dB.

4. *STD:* $\frac{S}{TD} \geq 37$dB.

5. *SNR:* $\frac{S}{N} \geq 37$dB.

6. *Channel Capacity:* $C > 34$Kbps.

[2] Technically speaking, most communication channels are causal and linear. For these systems, the EDD is also proportional to the slope of the attenuation curve.

3.3 Hybrid connections:

If one end is connected to the test line with 2-wire analog loop interface, while the other end is connected with digital PCM interface (such as the case when a 923 talks to a 950 or between two 930s through a channel bank), then the following numbers are commonly seen.

1. *Attenuations:* The attenuation curve will largely reflect the amplitude frequency response of the analog loop line and the characteristics of the band-pass antialiasing filter and the low-pass reconstruction filter used in a PCM line card. The attenuation curve should have either a low-pass filter characteristic (flat from DC to 3KHz, and roll off after 3KHz), or a band-pass characteristic (flat between 300 and 3300Hz, and roll off at the edges).

2. *EDDs:* In the same token, the EDD curve should reflect the phase response characteristics of the analog loop line and the PCM line card filters. The EDD curve should have a shape resembling the upside down mirror image of the attenuation curve.

3. *IMDs:* $IMD_2 > 45$ dB and $IMD_3 > 45$ dB.

4. *STD:* $\frac{S}{TD} \geq 36$dB.

5. *SNR:* $\frac{S}{N} \geq 36$dB.

6. *Channel Capacity:* $C > 34$Kbps.

There has been some suspicion that the digital side and analog side will observe different results. But there is no 'scientific' evidence for that. Both ends should have symmetric results within the measuring precision.

3.4 Test through a real local loop

When testing through a real local loop line and the switch in CO, the following results are common:

1. *Attenuations:* The attenuation curve will largely reflect the band-pass nature of the amplitude frequency response of a typical local loop.

2. *EDDs:* The EDD curve should reflect the phase response characteristic of a typical local loop, and have a shape resembling the upside down mirror image of the attenuation curve.

3. *IMDs:* $IMD_2 > 45$ dB and $IMD_3 > 45$ dB.

4. *STD:* $\frac{S}{TD} \geq 33$dB.

5. *SNR:* $\frac{S}{N} \geq 33$dB.

6. *Channel Capacity:* $C > 28$Kbps.

4 How to Perform the Precision and Impairment Tests

This section is particularly useful for test engineers to check the precision and accuracy of the 23-tone test.

4.1 Attenuation and EDD

The accuracies of attenuation and EDD distortions are quite straight forward to verify by selecting some typical impaired lines from TAS, and compare the TAS attenuation and EDD curves with the results measured by the 23-tone test.

When adding noise into the test line, the attenuation and EDD measurements should not be affected, unless the $\frac{S}{TD}$ is less than 25 dB. Neither should 2nd order IMD affect the attenuation and EDD measurements. But the 3rd order IMD will affect the attenuation and EDD measurements.

The precision requirement for attenuation is ±0.1dB, and the EDD precision is $\pm 10\mu s$.

4.2 IMDs

The IMD measurement accuracies can be verified by introducing IMDs through TAS. If everything is set up correctly (input and output level in TAS are set to match the transmit test signal level, for example), then the following relations are true:

$$\begin{aligned} IMD2_{23-tone} &= IMD2_{TAS} - 3dB \\ IMD3_{23-tone} &= IMD3_{TAS} \end{aligned}$$

The 3 dB difference arises from the fact that TAS introduces IMD calibrated to the 4-tone IMD test model. In 4-tone IMD test, the second order IMD power is 'arbitrarily' halved, therefore causing the 3 dB difference.

If one is not sure about the TAS set up, then one can check the 23-tone IMD measurements against the 4-tone IMD test. The IMD2s should be offset by 3 dB, and IMD3s should be the same.

When introducing pure additive noise or attenuation distortion, the IMD measurements should not be affected unless the SNR is less than 25 dB.

4.3 STD and SNR

The SNR measurement can be verified by introducing noise into the test line through TAS, and see if the measured SNR matches the theoretically predicted SNR. If the noise is introduced at the transmitting side, then the theoretical SNR is:

$$SNR = XMTLevel_{dBm} - NoisePower_{dBm} - FilterCorrection_{dB}$$

If the noise is introduced at the receiving side, then the theoretical SNR is:

$$SNR = RCVLevel_{dBm} - NoisePower_{dBm} - FilterCorrection_{dB}$$

In the above two equations, the $FilterCorrection$ is to account for the difference between various filters used in measuring noise. TAS introduces noise calibrated to C-message filter. The old-version of Sage's 23-tone test (the one that runs in 950 now) uses a box-car type of filter when weighting noise. The new 23-tone test (the one that runs on 923 and 356E+) cascades the IEEE 743 recommended D-filter and the box-car filter. For the old version of 23-tone code, $FilterCorrection = 2.1$dB. For the new version of 23-tone code, $FilterCorrection = 1.3$dB.

Notice that SNR only measures the non-correlated noise component. Introducing pure IMDs should not affect the SNR reading, unless significant amount of IMD3 is introduced and the composite signal power is affected.

STD measures the composite effect of additive noise and the IMDs. When there is no IMD, STD should equal SNR. When there is no noise but pure IMD, then STD should equal the combination of IMD2 and IMD3. Mathematically, STD can be calculated from SNR, IMD2 and IMD3 in the following fashion:

$$STD = -10log(10^{-0.1SNR} + 10^{-0.1IMD2} + 10^{-0.1IMD3})$$

4.4 Channel Capacity

This is the only 'wild card' that Sage cannot test in house reliably, and there are also confusions about what the measurement really means.

If the Channel Capacity measurement is strictly interpreted as a measurement based on Shannon-Hartley channel capacity theorem, then there is no need to test it, other than that the developer needs to make sure the numerical integration process is handled correctly. This can be checked by capturing some data and performing off-line calculation and see if the off-line channel capacity measurement matches the real-time calculation.

We should resist the temptation of relating the channel capacity to an actual modem rate. Doing so (or telling customers that we can do so) may drag Sage into endless troubles.

In short, the Channel Capacity measurement should be considered as a summary of the 23-tone test results. It shall not be considered as the actual modem rate. If the customer is fully aware of this fact, then there is no need to specifically test this item. If we want to make a modem rate predictor by stretching the Channel Capacity test, then Sage has a lot of tests to do.

5 Anomalies and Peculiarities

The composite 23-tone signal, and each of its 23 component all have an exact period of 64 ms. Furthermore, all the frequency components are odd integer multiples of $\frac{1000}{64} = \frac{125}{8}$. The design of this test signal is strictly for the convenience of DSP algorithm, so that all measurements can be based on 64 ms long FFT. The convenience of course comes at a price: the IMD3 anomaly over symmetric quantizer and its extreme sensitivity to sample jitter.

5.1 IMD3 anomaly

When conducting 23-tone test through clean PCM channel with digital connections, anomalously high IMD3 has been observed. The cause of the anomaly is clear by considering the following factors:

Synchronized transmitter and receiver: When both ends are connected with PCM digital interface, the receiving side is seeing the same 'bit-exact' quantization noise as introduced at the transmitting side. This situation is only true when both sides are digitally connected and synchronized through the T1 trunk. With analog connection, the transmitter and receiver synchronization (meaning the transmitter and receiver are locked into the exact same sampling points) is extremely unlikely (if you can, you can make a super-fast modem).

Symmetric PCM Quantizer: The quantization noise introduced by PCM logarithmic companding is symmetric. By 'symmetric', we mean if the original signal is symmetric, then the quantized signal carries the same symmetricity. Mathematically speaking, assume the quantized signal of $x(t)$ is $Q[x(t)] = x(t) + \epsilon(t)$, then a symmetric quantizer has the following 'trivial' property:

$$Q[-x(t)] = -Q[x(t)]$$

Odd Symmetric 23-tone Signal: The first half period of the 23-tone signal is odd symmetric relative to the second half period. Assume the period of the 23-tone signal is T (T=64 ms, in this case), then the following is true about the 23-tone signal:

$$x(t + \frac{T}{2}) = -x(t)$$

The above equation is true because the 23-tone signal is composed of 23 components whose frequencies are all odd integer multiples of the fundamental frequency ($F = \frac{1}{T}$).

Uneven Distribution of Quantization Noise: When passing the 23-tone signal through a symmetric quantizer, then the quantized signal retains the original signal's odd symmetricity. That is:

$$Q[x(t + \frac{T}{2})] = Q[-x(t)] = -Q[x(t)]$$

The implication of the above equation is that the quantized signal $Q[x(t)]$ retains the same odd symmetricity as the original 23-tone signal does. By properties of Fourier series expansion, the quantized signal $Q[x(t)]$ can only contain components whose frequencies are odd integer multiples of the fundamental frequency ($F = \frac{1}{T}$) as imposed by the odd symetricity property. Notice that the IMD3 measure the power at odd frequency bins, whereas IMD2 measure power at even frequency bins. When the above situations are true, one would measure $IMD_2 = \infty$ and $IMD_3 \approx 43$dB.

Undoubtedly, a clean PCM channel should have no IMD distortion ($IMDs \geq 50$dB). The 43 dB IMD3 is clearly an anomaly of the 23-tone test. Realizing this fact, Sage's 23-tone test algorithm specifically suppresses this IMD3 anomaly while retaining the accuracy of true IMD3 measurement.

5.2 Sensitivity to sample jitter

Notice that both the composite 23-tone signal and each of its components all have a period of 64 ms. This is convenient for DSP algorithm design so that a 64-ms long FFT can resolve all the relevant frequency bins. If the periodicity of the signal is destroyed by sample jitter (dropping or insertion of one or more data samples), the 23-tone measurement algorithm will measure anomalously high STD, SNR and IMDs. The cause is spectral leakage due to the lack of periodicity. To better illustrate the problem, Figure 1 shows three spectrum plots of the 'same clean' 23-tone signal, except two of them have one data sample being dropped.

As clearly shown in Figure 1, a single data sample shrinkage will cause significant rise of 'noise' floor, therefore causing erroneous readings on STD and SNR and even IMDs.

The scenario in Figure 1 will happen when the digital side clock (internal or loop-time clocks) is set up incorrectly. We have repeatedly seen this scenario when testing 930s or 923 with 950. In field applications, if occasional erroneously high STD is observed, the clock setting should be examined as a potential cause.

6 DSP Algorithms

This section details the 23-tone measurement algorithms. The sampling rate is assumed to be 8KHz so that a period of 64 ms corresponds to 512 samples.

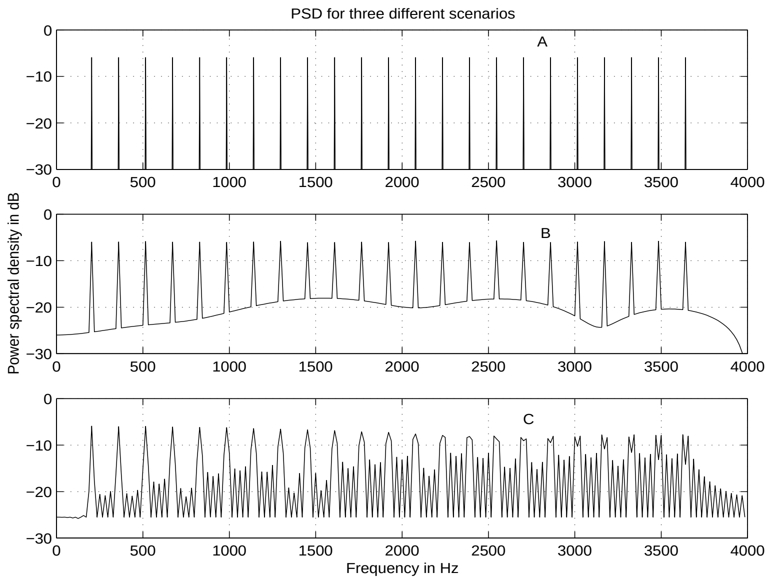

Figure 1: Spectral plots of the 23-tone signal with and without sample jitter. Case A: Spectrum of an ideal 23-tone signal. Case B: Spectrum of a jittered 23-tone signal where the 11th sample (8KHz sampling rate) is dropped (shrinked). Case C: Spectrum of a jittered 23-tone signal where the 256th sample is dropped (shrinked).

6.1 Bin notations

When performing 23-tone measurement, there are different groups of frequency bins that need to tracked. These groups are:

1. $s23bins$: This designates the original 23-tone signal bins. That is,

$$s23bins(i) = 10i + 13, i = 0, 1, \ldots, 22$$

2. $imd2bins$: This tracks the 42 IMD2 bins. That is,

$$imd2bins(2i) = 10i + 20, imd2bins(2i + 1) = 10i + 26, i = 0, 1, \ldots, 20$$

3. $imd3bins$: This tracks the 40 IMD3 bins. That is,

$$imd3bins(2i) = 10i + 17, imd3bins(2i + 1) = 10i + 39, i = 0, 1, \ldots, 19$$

4. $oddnsbins$: This tracks the 48 odd noise bins used to compensate the IMD3 anomaly. They are

$$oddnsbins(i) = [19\ 29\ 10i + 21\ 10i + 15\ 217\ 227], i = 0, 1, \ldots, 21$$

6.2 Signal Generator and D-filter Weighting

The 23-tone signal is generated according to:

$$s(n) = A \sum_{m=0}^{22} H_m \sin(\frac{2\pi}{512}(10m + 13)n + \phi_m^0 + \phi_m^1), n = 0, 1, \ldots, 511 \quad (1)$$

where $\phi_m^0, m = 0, 1, \ldots, 22$ is the initial phase of each component as specified in IEEE 743 [3]. A is related to the total transmitting power (dBm) as:

$$A = \sqrt{\frac{23}{2} 10^{0.1CPwr}}$$

H_m is the amplitude compensation for our own transmitter hardware (some minor ripples), and ϕ_m^1 compensates the phase response of the transmitter hardware. That is to say, the transfer function of our transmitter hardware is assumed to be $\frac{1}{H_m e^{j\phi_m^1}}$.

The D-filter coefficients are obtained by transforming the pole-zero locations specified in IEEE 743 into the canonical transfer function prototype. After this, an impulse response of the filter is obtained (truncated to 512 points) and the frequency weighting curve (frequency response) is obtained by FFTing the impulse response. This procedure is the easiest to implement and takes the least amount of real-time.

6.3 Segmentation of incoming data samples

Assume the incoming data samples are $r(n), n = 0, 1, 2, \ldots$, the first task is to group the samples into consecutive 512-point long segments according to:

$$x_m(n) = r(n + 512m), n = 0, 1, 2, \ldots, 511; m = 0, 1, 2, \ldots, M - 1 \tag{2}$$

where n designates the sample index within each segment, and m designates the segment index. The upper limit M is related to the test duration as:

$$M = \lfloor \frac{TestDuration}{0.064} \rfloor \tag{3}$$

6.4 FFTs

512-point FFTs are performed on each of the data segment:

$$X_m(k) = \sum_{n=0}^{511} x_m(n) e^{-j2\pi \frac{nk}{512}}, k = 0, 1, \ldots, 256 \tag{4}$$

6.5 Averagings

Two types of averaging are performed. The first one is averaging the power-spectral-density segment by segment. That is:

$$X'_{PSD}(k) = \frac{1}{M} \sum_{m=0}^{M-1} |X_m(k)|^2, k = 0, 1, \ldots, 256 \tag{5}$$

The goal of this averaging is to obtain a smooth estimation of the power-spectrum of the noise background.

The second averaging is performed on the complex spectrum directly:

$$X'_{CSD}(k) = \frac{1}{M} \sum_{m=0}^{M-1} X_m(k), k = 0, 1, \ldots, 256 \tag{6}$$

[3] Strictly speaking, if ϕ_m^0 are defined as initial phases, then the sin() function should be replaced by cos() function. But since the original IEEE 743 document uses the sin() function, we will stick to this 'incorrect' usage as it does not have actual impact other than mathematical symbolic error.

Notice that the above averaging is equivalent to time-domain averaging because FFT is a linear transformation. The goal of this averaging is to remove non-correlated noise from the original signal bins and the IMD bins.

6.6 Receiver hardware compensation

The received signal spectrum needs to compensated to eliminate our own receiver hardware effect. Assume the receiver front end has a transfer function of $\frac{1}{H(k)e^{j\phi(k)}}$, then the averaged PSD should be compensated according to:

$$X_{PSD}(k) = X'_{PSD}(k)|H(k)|^2, k = 0, 1, 2, \ldots, 256 \tag{7}$$

The averaged complex spectrum should be compensated according to:

$$X_{CSD}(k) = X'_{CSD}(k)H(k)e^{j\phi(k)}, k = 0, 1, 2, \ldots, 256 \tag{8}$$

The above equations imply that theoretically, the compensation should be performed across all 257 frequency bins. In actual implementation, however, only the original 23-tone bins are compensated. The impact on noise and IMD measurements by not compensating the noise and IMD bins are minimal unless our hardware front end is very sloppy, which is fortunately not the case, thanks to our diligent hardware designers.

6.7 Composite power

The composite power (in dBm) is calculated from the averaged PSD:

$$CPwr = 10\log\left(\frac{2}{512^2}\sum_{i=0}^{22}\left[X_{PSD}(s23bins(i)) - \frac{X_{PSD}(s23bins(i)-1) + X_{PSD}(s23bins(i)+1)}{2}\right]\right) \tag{9}$$

An alternative (and intuitively better) approach in calculating composite power is to use the averaged complex spectrum:

$$CPwr_{dBm} = 10\log\left(\frac{2}{512^2}\sum_{i=0}^{22}|X_{CSD}(s23bins(i))|^2\right) \tag{10}$$

Although simpler, the above approach seems to be more susceptible to sampling clock drift. So if not sure about the hardware performance, it's better to use the PSD approach.

For later calculations of IMDs, STD and SNR, the un-scaled composite power is also calculated:

$$CP = \sum_{i=0}^{22}\left[X_{PSD}(s23bins(i)) - \frac{X_{PSD}(s23bins(i)-1) + X_{PSD}(s23bins(i)+1)}{2}\right] \tag{11}$$

and the D-weighted composite power is:

$$CP_D = \sum_{i=0}^{22}\left[\left(X_{PSD}(s23bins(i)) - \frac{X_{PSD}(s23bins(i)-1) + X_{PSD}(s23bins(i)+1)}{2}\right)|D(s23bins(i))|^2\right] \tag{12}$$

where $D(i)$ is the frequency response of the D-filter.

6.8 Attenuations

The attenuations (losses in dB) at each bin are calculated as:

$$Loss_i = 10\log(\frac{(256A)^2}{X_{PSD}(s23bins(i)) - \frac{X_{PSD}(s23bins(i)-1)+X_{PSD}(s23bins(i)+1)}{2}}) \tag{13}$$

An alternative approach (if there is no sampling clock drift) is to use the averaged complex spectrum:

$$Loss_i = 10\log(\frac{(256A)^2}{|X_{CSD}(s23bins(i))|^2}), i = 0, 1, \ldots, 22 \tag{14}$$

In both of the above equations, A is defined in the signal generator section.

6.9 EDDs

Calculating EDDs involves several steps:

1. Obtain the 23 complex numbers: $Z(i) = X_{CSD}(s23bins(i)), i = 0, 1, \ldots, 22$.
2. Cancel the initial phase effect: $Zc(i) = Z(i)e^{j(\frac{\pi}{2}-\phi_i^0)}$.
3. Calculate phase difference by complex conjugate multiplication: $Zd(i) = Zc(i+1)Zc^*(i), i = 0, 1, \ldots, 21$, where the z^* stands for the conjugation of the complex number z.
4. Find the specific bin k such that $|Zd(k)|$ is the greatest compared with other $|Zd(i)|$. Move the whole complex constellation $Zd(i)$ around so that $Zd(k)$ coincides with the real-axis. This step is to guarantee the branch-cut does not cut through the constellation when taking principle phase value from the complex numbers. This step is obtained by $Ze(i) = Zd(i) \times Zd^*(k), i = 0, 1, \ldots, 21$.
5. Obtain the EDDs: The EDDs are calculated by inverting the phase from the complex numbers and scale them:
$$EDD(i) = -\frac{3200}{\pi}\arctan(Ze(i)), i = 0, 1, \ldots, 21 \tag{15}$$
where the unit for EDDs is μs. The principle values of the arctan() function are within $[-\pi, \pi]$.
6. Offset removal and smoothing: The minimal value of EDDs is first found, and then subtracted from all the EDDs so that the EDDs will be all positive and the minimal value becomes zero. If the difference between two adjacent bins are greater than 3200, then it will be wrapped around by 6400.

6.10 IMDs

The powers of IMD2 and IMD3 are calculated from the averaged complex spectrum:

$$P_{IMD2} = \sum_{i=0}^{41} |X_{CSD}(imd2bins(i))|^2$$
$$P_{IMD3} = \sum_{i=0}^{39} |X_{CSD}(imd3bins(i))|^2 \tag{16}$$

The so-called IMDs in dB are then calculated as:

$$IMD_2 = 10\log(\frac{CP}{P_{IMD2}})$$
$$IMD_3 = 10\log(\frac{CP}{P_{IMD3}}) \tag{17}$$

where CP is calculated in the composite power section.

6.11 STD and SNR

To calculate STD and SNR, the following components need to be calculated:

1. Obtain the D-weighted total distortion from X_{PSD}:

$$TD = [\sum_{i=13}^{233} X_{PSD}(i)|D(i)|^2] - CP_D \tag{18}$$

   where CP_D is calculated in the composite power section.

2. Obtain the D-weighted IMD powers from X_{CSD}:

$$IMD = \sum_{i=0}^{41} |X_{CSD}(imd2bins)D(imd2bins(i))|^2 + \sum_{i=0}^{39} |X_{CSD}(imd3bins)D(imd3bins(i))|^2 \tag{19}$$

3. The D-weighted total noise power is:

$$NP = TD - IMD \tag{20}$$

The STD and SNR (in dBs) are then calculated as:

$$STD = 10\log(\frac{CP_D}{TD})$$
$$SNR = 10\log(\frac{CP_D}{NP}) \tag{21}$$

6.12 Channel Capacity

The channel capacity is calculated as:

$$CC_{bps} = \sum_{m=0}^{22} \log_2(1 + \frac{X_{PSD}(s23bins(m))}{[\sum_{k=0}^{9} X_{PSD}(s23bins(m)+k-5)] - X_{PSD}(s23bins(m))}) \tag{22}$$

Work is still going on to make this calculation more sophisticated so that it can be directly related to the actual V.90 modem rate. Until we see some in-house testings, it is not appropriate to fully disclose details here.